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# Narrow Strip Above Ground Plane Transmission Line Formulation in the FDTD algorithm

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## 1. Introduction

A number of thin wire formalisms for use in the FDTD method have been published over the years. One such formalism was published by Holland and Simpson [1] in 1981. Later, Ledfelt, [2], enhanced the method in order to improve the accuracy for arbitrarily positioned wires by using a "shell average" in order to derive the average electric field around the wire. This approach was further developed by Edelvik, [3], who introduced a newly defined basis function to represent the current density around the wire. Very recently Koh [4] has extended this approach to allow wire transmission line problems to be treated.

In this paper the methods used in [4] are, for the first time, applied to the treatment of narrow strips with various terminations. It is shown that, with the correct choice of in-cell inductance and the appropriate use of tri-linear distribution and shell-average interpolation, accurate results can be obtained without the need for a fine FDTD mesh.

## 2. The formulation

The method of Holland and Simpson [1] and its derivatives make use of auxiliary differential equations which relate the current on the wire to the charge and the surrounding tangential E field. For instance equation 10 in [1] can be written as:

$$L \left( \frac{\partial I}{\partial t} + \frac{1}{\epsilon_0 \mu_0} \frac{\partial Q}{\partial z} \right) = \langle E_z \rangle \quad (1)$$

where  $I$  is the wire current,  $Q$  is the wire charge per unit length,  $\langle E_z \rangle$  is the averaged electric field surrounding the wire and  $L$  is a constant having the dimensions of inductance and generally referred to as the in-cell inductance. This is calculated making use of the known static field distribution around the wire and depends on the radius of the wire and the FDTD cell size.

In order to apply the method to strips, it is necessary to ascertain the appropriate value for  $L$ . This is readily done by using the known static field distribution around a strip[5]. The rest of the formulation proceeds as before. It is shown in [6] that the appropriate value is given by equation (2) where,  $\Delta$ , is the FDTD cell size and,  $w$ , is the strip width.

$$L = \frac{\mu}{2\pi} \operatorname{Re} \ln \left( \frac{w/2}{i\Delta + \sqrt{(i\Delta)^2 + \left(\frac{w}{2}\right)^2}} \right) \quad (2)$$

As in [4], the singular nature of the fields is taken into account when formulating the FDTD update equations for the transverse electric field components and the circulating magnetic field components.

At a lumped element termination, it was found necessary to use a tri-linear distribution for the currents and the shell-averaged interpolation, described in [2], for the E field.

### 3. Simulation validation

To verify the accuracy of the proposed algorithm an initial trial was done to calculate the characteristic impedance of a strip placed 10mm above a ground plane for various strip widths. To confirm consistency of results, two different test geometries were used. These are the same as were used in [4] except that the test object is a strip instead of a wire.

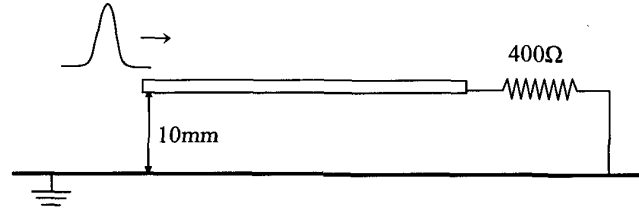


Figure 1 - Geometry of the test structure

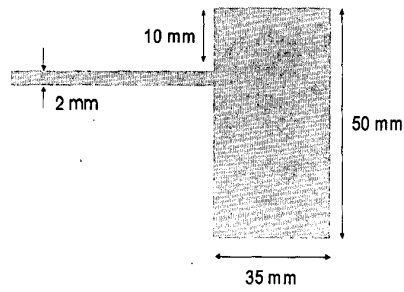
In the first case the strip is terminated with a 400Ω resistive load as shown in Figure 1. In the second case the strip is terminated with a transmission line having a characteristic impedance of 236Ω. The FDTD domain was 0.1m x 0.1m x 1m divided into uniform cells of 2mm. The excitation was a 1.5ns raised cosine pulse. In addition, the characteristic impedance was calculated in two different ways. Firstly by recording the amplitudes of the incident and reflected pulses to and from the termination, and secondly by examining the E and H field distribution around the strip. The results are summarised in Table 1 where the percentage discrepancy between the predicted results and results provided by a standard quasi-static formula are given.  $\Gamma$  is the calculated reflection coefficient from the termination.

**Table 1 - Errors in predicted values of the characteristic impedance,  $Z_0$ , of a strip over a ground plane**

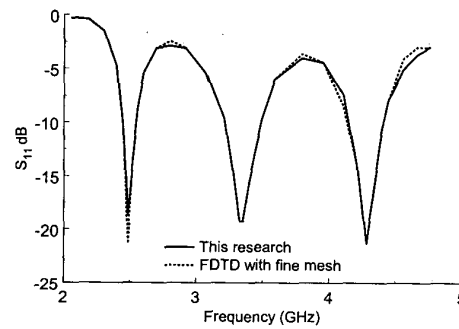
Strip Width (mm)	$Z_0$ from formula[7] ( $\Omega$ )	400 $\Omega$ resistor		236 $\Omega$ transmission line	
		% error in $Z_0$ from fields	% error in $Z_0$ from $\Gamma$	% error in $Z_0$ from fields	% error in $Z_0$ from $\Gamma$
0.5	305	1.7	1.9	1.6	1.6
0.25	346	1.3	1.5	1.5	1.5
0.1	401	1.0	1.0	1.0	1.0

This confirms that, using the appropriate value of in-cell inductance together with the appropriate combination of tri-linear distribution and shell interpolation, accurate values for the characteristic impedance of strip line can be obtained without the need for a very fine mesh.

The method was also applied to the prediction of the return loss of a patch antenna with a feedline of width 2mm. The geometry is given in Figure 2. The height of the patch above the ground plane is 7.5mm.



**Figure 2 - Geometry of the patch antenna**



**Figure 3 - Calculated return loss using different methods**

In Figure 3, the predicted result using the method proposed in this paper is compared to a reference result calculated using standard FDTD with a much finer mesh. It can be seen that the results are in very good agreement indicating that the method described here enables good results to be obtained without the need for a fine mesh.

#### 4. Conclusions

An extended form of Holland's thin wire formulation has been successfully adapted to allow for the treatment of terminated narrow strips. The proposed method has been verified by demonstrating that the predicted characteristic impedance of a terminated strip above a ground plane is in good agreement with a standard formula and by demonstrating that the correct return loss for a patch antenna is predicted.

These results show both that the strip itself is being correctly modelled and also that the discontinuity at the termination is being appropriately treated both for the case of a lumped resistor and for a patch.

Work is in progress to apply this method to more complex microstrip circuits and Printed Circuit Boards.

#### 5. References

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